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PROTON STOPPING POWER AND ENERGY STRAGGLING OF PROTONS

BY

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Introduction.

The energy loss of protons in foils of various substances has been determined as a function of the proton energy. The sharp resonances for proton reactions have been used for analyzing the energy distribution of a beam of protons after its passage through a thin layer of various substances. Because of the energy loss in such a layer, the resonance curves are found at a higher voltage when a foil is inserted in the beam than when no foil is inserted. The shift of proton energy gives the energy loss in the foil.

Moreover, a broadening of the resonance curves obtained when a foil is inserted in the beam makes it possible to determine the energy straggling. Some measurements, concerning mainly beryllium and mica, have been published earlier[1),2),3)]. In the present paper, experimental results are reported for other materials of higher atomic numbers and the range of proton energy is extended to 2 MeV.

Experimental.

The experiments were carried out at the Institute for Theoretical Physics in Copenhagen with the pressure insulated van de Graaff generator[4)]. The rotating compensation voltmeter was calibrated by the proton capture resonance in aluminum at 503 keV and the linearity was checked by measuring the 503 keV resonance with protons and molecular ions.

The proton current was of the order of 2 μA when no foil was inserted in the beam; when a foil was used the current was adjusted to about 0.2 μA. The current integrator[5)] consists of a recorder, which counted the pulses of a neon lamp, discharging a condensor constantly charged by the target current. At the lowest energies where the stopping power and, consequently, the

heating of the foil are greatest, and especially for thicker foils, the integrator could not be used. In such cases, the beam current was measured with a sensitive galvanometer and kept constant by means of small variations of the voltage of the probe in the source.

As γ-ray counter a Geiger-Müller tube, 10 mm in diameter and 40 mm long, was used. It was placed in a lead box as close to the target as possible and connected to a scale-of-32. The neutrons were detected by means of boron neutron counters surrounded by paraffin wax. An electronic switch[4] blocked the counting of both γ-rays and neutrons when the acceleration voltage differed by more than about 2 keV from the desired value. This blocking of the counting arrangement makes it possible to obtain sharper resonance peaks.

The foils were placed in a small disk with six circular openings, some of which were covered by foils. The disk was mounted in the acceleration tube at a distance of 35 cm from the target in order to reduce the background radiation from reactions in the foil as far as possible. The disk could rotate on an axis through its center, and all openings were placed in the same distance from the axis. With the aid of a magnet the different openings of the disk could be brought into the path of the protons. Foils of different thicknesses were used as listed in the tables. They were cut into small round pieces of a diameter of 11 mm after their thickness had been determined by measuring the weights and areas of larger pieces.

Because of the stop in the Faraday cage the scattering in the foil decreases the current to the target. Moreover, the straggling in the foil causes a broadening of the resonance peak and a consequent diminishing of the peak intensity. For these reasons only the strongest peaks could be used as energy indicators. The following peaks have been used: Proton-capture resonance, in fluorine at 339 and 660 keV[7], in aluminum at 630, 986, and 1255 keV[4], and in chlorine at 860 keV[8]. Due to the great density of the levels none of the capture processes can be used at higher potentials. The measurements about 2 keV are based on the (p, n) resonance at 1974 keV[8] in the process Cl^{35} (p, n) A^{35}. Measurements at the lowest proton energies were carried out with the molecular beam, since focussing was unstable at low

generator voltages; however, this should not influence the results, as the molecules are split up as soon as they hit the foil.

Targets were prepared by evaporation of calcium fluoride, aluminum or lead chloride in vacuum on small disks of silver or copper. Copper backings were used for γ-ray observations from proton capture reactions with a proton energy smaller than 0.9 keV, and for the (p, n) reaction at 2 MeV. Silver targets were used for γ-rays from capture reactions with a proton energy higher than 0.9 MeV. The small width of the resonance levels indicates that a target with a stopping power of 1—2 keV, giving saturation intensity, is suitable. However, the broadening of the peaks due to the energy straggling in the foil, especially at lower energies, necessitates a thicker target. Accordingly, many targets of different thicknesses were used.

Most of the measurements were performed on foils. Both commercial foils which are rolled out and foils prepared in the laboratory by evaporation in vacuum were used. The commercial foils are rather inhomogeneous and are only convenient for the determination of the stopping power, because the broadening of the resonance curve in these cases is not only due to the straggling in the foil. Foils produced by evaporation seem to be rather homogeneous. In a previous paper[3] the straggling in foils of beryllium and mica has been described.

Besides foils also sandwich targets have been used. They were prepared in the following way. On 3—4 disks of the support material (copper or silver) a layer, containing the energy indicator (fluorine, aluminum or chlorine), is evaporated. One or two disks are removed from the evaporation chamber, while the stopping substance (Be, Al, Cu, Ag or Bi) is evaporated on the energy indicator layer of the remaining targets*.

In this way, heating of the foil is avoided and a greater beam current can be used. Moreover, a decrease in the beam current caused by scattering is prevented. However, some difficulties arose from the determination of the thickness of the stopping layer. The increase in weight of the target caused by the stopping layer was often too small as to be measured with sufficient accuracy.

* Thanks are due civilengineer Mr. O. B. NIELSEN for preparing sandwich targets.

Before evaporation of the stopping substance, the targets were placed on pieces of glass. Both the targets and the glass around the targets were covered with stopping material. After removal of the targets the thickness of the stopping layer was found by measurements of the interference between the rays coming from the clean surface of the glass behind the target and the rays from the surface of the stopping layer. In many cases, however, the thicknesses found in this way disagreed with results obtained in other ways. This may be due to different reflections of the molecular particles from the metal surface of the target itself and from the surface of the glass.* Therefore the thickness of a stopping layer was determined by comparing the energy loss of the layer with that of a foil with known thickness. Thus, a value of the stopping power found by means of the sandwich targets is determined relative to measurement with foils. However, the principal value of such targets lies in their homogeneity and in the possibility of obtaining better resonance peaks for the determination of the straggling.

In order to find the energy loss ΔE and the straggling Ω in a foil or in a sandwich target, the energy distribution of the peaks is approximated by the Gaussian, even though this should not always be the correct shape, at least not for the peaks measured without foil or stopping layer, where the width is due mainly to the thickness of the target.

As it was shown in greater detail in the previous paper[3], the resonance curves are transformed into straight lines by means of tables[9] or probability paper. The energy loss ΔE is the difference between the energies corresponding to the mid point of the straight lines (probits = 5) found with and without a foil, respectively.

Since the standard deviations are added geometrically the true straggling Ω is found by means of the formula $\Omega = \sqrt{\Omega_2^2 - \Omega_1^2}$, where Ω_2 and Ω_1 are the standard deviations corresponding to the measurements with and without foil, respectively. These deviations can be found from the slopes of the straight lines.

ΔE and Ω are thus determined in a rather unambiguous way. In most cases, the resonance curves are symmetrical and

* The author is indebted to mag. scient. Rahbek for carrying out the interference measurements.

the transformed curves are straight lines. In such cases, the most probable energy loss and the mean energy loss are the same. By far the greatest number of the resonance curves could be transformed to straight lines. The shift could be found with an accuracy of less than 1 keV and the slope was determined accurate to ten per cent.

In some cases a transformed curve was not a straight line because of a long tail of the resonance curve. In such a case the most probable energy loss and the mean energy loss are not identical; however, the difference is small, about 1 keV. In comparison with a shift of 30—50 keV this uncertainty does not influence considerably the value of the stopping power. The value for the straggling is greatly affected by the deviations from a straight line. In a few of these cases, an attempt was made to determine the straggling by determining that Gaussian distribution which, folded with the resonance curve found without foil, gave the best fit to the curve obtained with a foil inserted in the proton beam. However, no greater accuracy could be obtained in this way. These values of the straggling may thus be rather uncertain (cf. the spread of the points on fig. 3).

Results.

The results obtained are presented in the following tables, which also include the values published in previous papers[1, 2, 3]).

The columns show:

1) Thickness t of the stopping material in mg per cm^2. The areas of the foils are calculated from measurements with a travelling microscope and their weights are determined with a microbalance. The thicknesses obtained in this way are believed to be correct within a few per cent. After the measurements the thicknesses were checked and found to be unchanged. When a sandwich target was used, further details are given in a footnote.
2) The energy shift ΔE in keV.
3) The proton energy E in keV. The energy given in the table is the mean value of the proton energies of the peaks of the resonance curves found with and without stopping material.

4) Stopping power of the substance measured in keV per mg per cm^2.
5) Standard deviation Ω_1 in keV, without stopping layer.
6) Standard deviation Ω_2 in keV, with stopping layer.
7) Straggling Ω in keV.

The straggling is determined neither for commercial foils nor for cases where the resonance peak is too small to give a tolerable accuracy in the value for the standard deviation.

Beryllium. Only foils prepared by evaporation have been used and the total contamination of other metals was found, by spectral analysis, to be less than 0.1 per cent.

In a previous paper[1] two curves found without and with foil are shown.

TABLE I. *Beryllium.*

t thickness	ΔE shift	E proton energy	s stopping power	Ω_1 standard deviations	Ω_2	Ω straggling
mg/cm²	keV	keV	keV/mg/cm²	keV	keV	keV
1	2	3	4	5	6	7
0.609	230	455	377	4.2	9.5	7.2
0.222	73	540	329	4.4	6.1	4.2
0.222	62	661	279	1.5	5.1	4.9
0.222	64	662	288	4.2	5.3	3.3
0.245	69	665	282	—	a)	—
0.222	53	798	239	—	a)	—
0.222	48	1010	216	1.4	5.0	4.8
0.222	48	1010	216	1.7	4.2	3.9
0.245	53	1013	216	1.3	4.2	4.0
0.609	126	1049	207	2.5	6.6	6.1
0.222	42	1133	189	2.9	5.3	4.5
0.222	45	1135	203	2.9	5.5	4.7
0.222	42	1276	189	1.7	4.9	4.6
0.609	108	1310	177	—	b)	—
0.222	39.5	1392	178	—	b)	—
0.609	101	1422	166	—	b)	—
0.610	85	2016	139	4.2	8.2	6.6

a) the intensity of the resonance is too low to determine Ω_2.
b) the two peaks Al 1372 and Al 1379 are superposed, Ω_2 cannot be determined.

Mica. The foils were prepared by splitting mica into thin pieces. On account of the optical transparency of a mica foil, information about the homogeneity can be obtained by interferometer measurements, according to the method of Tolansky. Details of these measurements are published in the earlier paper[3]. Thus, only for the mica foils it has been possible to check the homogeneity directly.

TABLE II. *Mica*.

t thickness	ΔE shift	E proton energy	s stopping power	Ω_1 standard deviations	Ω_2	Ω straggling
mg/cm²	keV	keV	keV/mg/cm²	keV	keV	keV
1	2	3	4	5	6	7
0.336	101	389	300	3.6	5.8	4.5
0.336	75,4	668	225	3.4	5.6	4.4
0.336	59	1016	176	1.7	5.2	4.9
0.336	59	1016	176	1.7	5.0	4.7
0.441	76	1024	172	1.6	6.4	6.2
0.441	76	1024	172	1.6	6.1	5.9
0.441	76	1024	172	1.6	5.9	5.7
0.441	76	1024	172	1.5	6.3	6.1
0.665	111	1041	168	1.7	7.6	7.4
0.665	113	1042	171	1.7	8.0	7.8
0.73	115	1047	158	3.5	7.2	8.3
0.78	131	1051	168	3.7	7.7	6.8
0.76	130	1051	171	3.5	8.0	7.2
0.76	130	1051	171	3.6	6.7	5.7
1.02	169	1070	166	1.9	9.4	9.2
1.02	173	1072	170	2.1	9.5	9.3
1.02	172	1072	168	2.1	9.1	8.9
1.02	172	1072	168	1.9	9.0	8.8
1.02	175	1073	171	1.7	8.7	8.5
1.23	212	1092	172	3.0	9.5	9.0
1.71	284	1140	166	3.8	12.5	11.9
1.06	155	1180	146	2.5	9.5	9.2
0.336	48	1279	143	3.0	4.8	3.8
0.336	36	1992	107	4.8	5.9	3.5
0.730	79	2014	108	4.8	8.5	7.0

Aluminum. In most measurements commercial foils were used. The total contamination of other substances was found

by chemical analysis to be less than 1 per cent*. No value for the straggling is determined for the commercial foils. Sandwich

TABLE III. *Aluminum.*

t thickness	ΔE shift	E proton energy	s stopping power	Ω_1 standard deviations	Ω_2	Ω straggling
mg/cm²	keV	keV	keV/mg/cm²	keV	keV	keV
1	2	3	4	5	6	7
0.23 a)	58	368	252	—	—	—
0.38 a)	95	387	250	—	—	—
0.23 a)	42	681	182	—	—	—
0.38 a)	70	695	185	—	—	—
0.23 a)	38.5	1005	167	—	—	—
0.38 a)	59.5	1016	157	—	—	—
0.48	72	1020	150	3.5	6.9	6.0
0.50	82	1025	164	3.5	7.0	6.0
0.23 a)	31	1270	135	—	—	—
0.38 a)	53	1281	139	—	—	—
0.23 a)	23	1986	100	—	—	—
0.38 a)	37	1993	98	—	—	—
0.37 b)	37	1993	100	4.8	7.2	5.4
0.37 b)	39	1994	105	4.8	6.2	4.0

a) commercial foil.
b) sandwich target. Thickness found from the width of the (p, γ) resonance radiation of the stopping layer at 986 keV.

targets can only be used at the (p, n) resonance of chlorine at 1974 keV because the aluminum itself will give rise to disturbing radiation if (p, γ) reactions are used as energy indicators.

Copper. Only commercial foils and two sandwich targets have been used. The contamination of other substances was found by chemical analysis to be about 5 per cent. As, however, the main part of the other substances was zinc, whose atomic number differs only by one unit from that of copper, this contamination does not influence the results appreciably.

* Thanks are due amanuensis T. LANGVAD for carrying out the analysis.

TABLE IV. *Copper.*

t thickness	ΔE shift	E proton energy	s stopping power	Ω_1 standard deviations	Ω_2	Ω straggling
mg/cm²	keV	keV	keV/mg/cm²	keV	keV	keV
1	2	3	4	5	6	7
0.49 a)	73	376	149	—	—	—
0.58 a)	90	384	155	—	—	—
0.49 a)	60	690	122	—	—	—
0.58 a)	70	695	121	—	—	—
0.59 b)	64	892	—	—	—	—
0.84 b)	91	905	—	2.5	6.5	6.0
0.49 a)	53.5	1012	109	—	—	—
0.50 a)	53.0	1012	106	—	—	—
0.58 a)	56.5	1014	98	—	—	—
0.49 a)	46	1278	94	—	—	—
0.58 a)	51	1281	88	—	—	—
0.49 a)	35	1991	72	—	—	—
0.58 a)	46	1997	79	—	—	—
0.59 b)	41.4	1994	70	5.4	7.5	5.2
0.84 b)	54	2001	64	4.8	8.1	6.5
0.84 b)	56	2002	67	4.3	7.0	5.6

a) commercial foil.
b) sandwich target. (Chlorine as indicator) thickness determined from the shift of the (p, γ) resonance at 860 keV.

Silver. Commercial foils, foils prepared by evaporation, and sandwich targets were used. The contamination was found by chemical analysis to be less than 1 per cent.

TABLE V. *Silver.*

t thickness	ΔE shift	E proton energy	s stopping power	Ω_1 standard deviations	Ω_2	Ω straggling
mg/cm²	keV	keV	keV/mg/cm²	keV	keV	keV
1	2	3	4	5	6	7
0.432 b)	59	369	137	4.8	6.3	4.1
0.42 a)	60	370	143	—	—	—
0.42 a)	60	370	143	—	—	—
0.36 a)	51	365	142	—	—	—

(continued)

TABLE V. *Silver.* (continued)

t thickness	ΔE shift	E proton energy	s stopping power	Ω_1 standard deviations	Ω_2	Ω straggling
mg/cm²	keV	keV	keV/mg/cm²	keV	keV	keV
1	2	3	4	5	6	7
0.59 a)	84	381	142	—	—	—
1.51	209	445	138	3.8	8.9	8.0
0.225 c)	22.8	641	101	2.2	3.7	3.0
0.235 c)	22.6	641	96	2.1	3.6	2.9
0.285 c)	32.5	646	114	2.7	4.7	3.8
0.36 a)	36	678	100	—	—	—
0.432 b)	45	682	—	5.5	7.0	4.4
0.442 a)	44	683	100	—	—	—
0.59 a)	64	692	108	—	—	—
0.51 e)	47.5	880	93	2.2	6.0	5.6
0.81 d)	75	885	—	—	—	—
0.225 c)	18.6	995	—	2.1	3.7	3.1
0.235 c)	19.5	996	—	2.0	3.7	3.1
0.285 c)	24.4	998	—	2.5	4.3	3.5
0.434 c)	36.2	1002	—	2.4	6.2	5.7
0.36 a)	33	1003	91	—	—	—
0.42 a)	34.7	1004	82	—	—	—
0.48 a)	38.5	1005	80	—	—	—
0.49 a)	40	1006	82	—	—	—
0.59 a)	53	1012	90	—	—	—
1.51	130	1051	86	3.4	8.0	7.2
0.225 c)	15.9	1263	71	2.3	3.8	3.0
0.225 c)	15.9	1263	71	1.8	3.2	2.7
0.434 c)	29.6	1270	68	2.3	4.3	3.6
0.28 e)	23	1986	—	6.0	8.3	5.7
0.49 a)	29	1989	59	—	—	—
0.51 e)	30	1989	—	3.8	6.8	5.6
0.81 d)	47.5	1998	59	6.5	7.7	4.1
0.91 e)	66	2007	—	6.0	8.7	6.3
1.51	90	2019	60	—	—	—

a) commercial foil.

b) sandwich target, fluorine as indicator, thickness determined from the shift of the (p, γ) resonance at 660 keV.

c) sandwich target, aluminum as indicator, thickness from the shift of the (p, γ) resonance at 986 keV.

d) sandwich target, chlorine as indicator, thickness from the shift of the (p, γ) resonance at 860 keV.

e) sandwich target, chlorine as indicator, thickness from the shift of the (p, n) resonance at 1974 keV.

Gold. Commercial gold foils have been used. As the production of sandwich targets covered with gold did not succeed, sandwich targets covered with bismuth were used. The stopping power varies with $Z^{\frac{1}{2}}$ and thus a value of the stopping power found by means of a sandwich target covered with bismuth must be increased by 2 % when compared with the stopping power of gold. When the thickness of a sandwich target is found from the shift of a resonance curve this factor must also be taken into account. In the table the measured stopping powers

TABLE VI. *Gold.*

stopping substance	t thickness	ΔE shift	E proton energy	S_{Bi}	S_{Au} stopping powers	Ω_1	Ω_2 standard deviations	Ω straggling
	mg/cm²	keV	keV	keV/mg/cm²		keV	keV	keV
1	2	3	4	5	6	7	8	9
Au	0.45 a)	38.5	359	—	85	—	—	—
Au	0.52 a)	42	360	—	81	—	—	—
Bi	0.55 b)	44	366	80	82	4.8	6.0	3.6
Bi	0.61 c)	41.9	651	69	71	2.1	5.0	4.6
Bi	0.61 c)	42.9	652	70	72	2.1	5.3	4.9
Bi	0.66 c)	49.4	655	74	76	2.3	6.8	6.4
Bi	0.55 b)	38	679	—	—	4.6	7.2	5.5
Bi	0.69 d)	42.3	881	—	—	2.5	6.0	5.5
Bi	0.69 d)	43.3	881	—	—	2.5	5.9	5.4
Au	0.45 a)	27	999	—	60	—	—	—
Au	0.52 a)	30	1000	—	58	—	—	—
Au	0.51 a)	31	1001	—	61	—	—	—
Bi	0.61 c)	35.4	1004	—	—	2.2	5.8	5.4
Bi	0.66 c)	38.7	1005	—	—	2.0	6.0	5.7
Bi	0.68 c)	39.4	1006	—	—	2.2	6.1	5.7
Au	0.84 a)	49.5	1009	—	59	—	—	—
Bi	0.61 c)	29.6	1270	49	50	2.0	6.0	5.7
Bi	0.69 d)	29	1988	42	43	5.8	7.6	5.0
Bi	0.69 d)	28.7	1988	42	43	4.8	6.5	4.4
Au	0.84 a)	34	1991	—	41	—	—	—

a) commercial foil.

b) sandwich target, fluorine as indicator, thickness determined from the shift of resonance at 660 keV.

c) sandwich target, aluminum as indicator, thickness determined from the shift of resonance at 986 keV.

d) sandwich target, chlorine as indicator, thickness determined from the shift of resonance at 860 keV.

for bismuth are given in column 5, and in column 6 are given the two per cent higher values valid for the stopping power of gold.

The values found for the stopping power $\left(\text{in } \frac{\text{keV}}{\text{mg/cm}^2}\right)$ are plotted as a function of the proton energy in fig. 1. Also the results obtained by WARSHAW[10] at lower energies are plotted on the figure. It is seen that, for each substance, the points can be connected by a smooth curve.

Discussion.

The energy loss per cm path is given by BETHE's formula[11]:

$$-\frac{dE}{dx} = \frac{4\pi e^4 Z_1^2}{mv^2} \cdot NZ_2 \cdot \log \frac{2mv^2}{I}. \qquad (1)$$

Here, e and m are the electronic charge and mass, v is the velocity, and Z_1 the atomic number of the incident atom, N is the number of atoms per cm^3, Z_2 is the atomic number and I the average excitation potential of the stopping substance.

A relativistic treatment shows that another term has to be added to the logarithm. At the velocities used in the present investigation this term can be omitted. The formula (1) is only valid if the velocity of the incident particle is much higher than the velocity of the electrons of the stopping substance.

In a previous paper[1] concerning the stopping power of solid beryllium, the formula (1) has been used in the form

$$S = \frac{4\pi e^4}{Em} \cdot \frac{Z_2}{A_2} \cdot \left\{\log\left(\frac{4E}{I} \cdot \frac{m}{M}\right) - \frac{C_K}{Z_2}\right\}, \qquad (2)$$

where S is the stopping power for unit thickness, E the proton energy, M the proton mass, and C_K is a correction due to the strong binding of the K-electrons, which has been calculated by BETHE[10] for the lightest elements.

The value of I was found to be equal to 64 ± 5 eV, in good agreement with a theoretical estimate of A. BOHR[12], who gives I to be about 60 eV. Recently, BAKKER and SEGRÈ[13] and MATHER and SEGRÈ[14], using 340 MeV protons whose energies are so high

that the C_K-corrections may be omitted, found values of I very close to 60 eV.

For the heavier substances, the velocities of the most firmly bound electrons may be comparable with or exceed the velocity of the incident particles. As no C_K-corrections are available for the heavier elements and, moreover, corrections due to the L and M shells are also necessary, the formulas (1) and (2) no longer apply.

For such cases, N. Bohr[15], using a simplified atomic model, gives an approximate expression and finds that the specific energy loss is proportional to $Z_2^{\frac{1}{3}} \cdot v^{-1}$. This dependence on v is found to hold approximately in the present experiments, but the numerical value given by Bohr is about 1.5 times the experimental results.

Recently, Lindhard and Scharff[16] have calculated energy losses for lower velocities. By means of a statistical argument they derive the quantity $L(x) = (\Delta E \cdot mv^2/4\,NZ_2e^4\,\Delta R)$ as a common function of the variable $x = Z_2^{-1}\left(\frac{v}{v_0}\right)^2$ for heavier substances.

For higher velocities, Bloch[17] has obtained the formula $L(x) = \log(2mv^2/Z_2I_0)$, where I_0 is a constant.

In order to compare the measurements with these formulas, the stopping powers at 350, 650, 1000, 1500, and 2000 keV found from fig. 1 are given in table VII.

Table VII. Specific Stopping Power (keV/mg/cm^2).

Substance \ Proton energy	350	650	1000	1500	2000 keV
Beryllium	386	285	215	167	138
Mica	310	220	170	134	107
Aluminum	270	190	158	119	99
Copper	170	125	105	84	70
Silver	138	104	83	68	58
Gold	83	70	59	50	42

From these five values for each element (mica excluded) the experimental values of $L(x)$ have been found and are plotted

against $x^{\frac{1}{2}} = Z_2^{-\frac{1}{2}} \frac{v}{v_0}$ on fig. 2. On this figure, a straight line through the origin would correspond to a specific stopping power proportional to $Z_2^{\frac{1}{2}} \cdot \frac{1}{v}$; this is seen to hold, in first approximation,

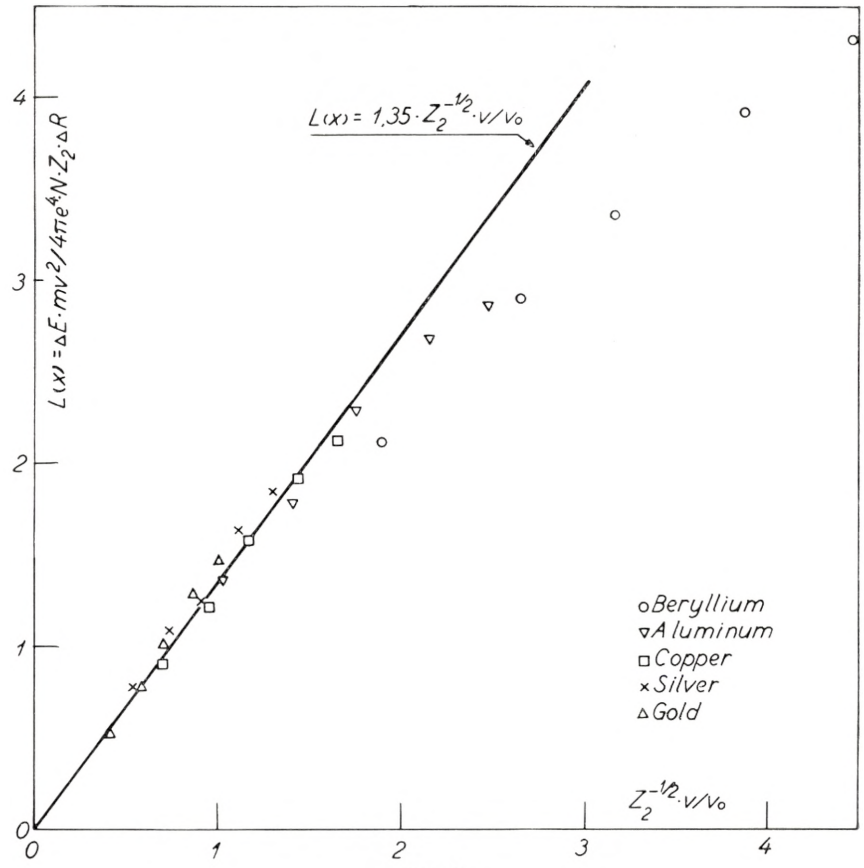

Fig. 2.

for the experimental points corresponding to the four heaviest elements.

The experimental points of beryllium do not fit with a straight line, because the velocities of the protons are so high that the formula of BLOCH is valid. The value of the Bloch constant $\frac{I}{Z_2}$ is 16 ± 1 eV in agreement with the results of BAKKER and SEGRÈ[13].

Several authors have tried to find an empirical formula for the variation of the stopping power with the atomic number Z. BRAGGS and KLEMANS[19] assumed that the stopping power S was proportional to $A^{\frac{1}{2}}$, where A is the mass number. RAUSCH v. TRAUBENBERG[18] adopted $S \cdot Z_2^{-\frac{1}{2}} = \text{const.}$ and GLASSON[20] found that $S \cdot Z_2^{-\frac{2}{3}} = \text{const.}$ gave the best fit.

The formula of BOHR[15] and that of LINDHARD and SCHARFF[16] may be written

$$L(x) = k_1 \cdot Z_2^{-\frac{2}{3}} \cdot \frac{2v}{v_0} \tag{3}$$

and

$$L(x) = k_2 \cdot Z_2^{-\frac{1}{2}} \cdot \frac{v}{v_0}. \tag{4}$$

From the values of $L(x)$ used on fig. 2 and from corresponding values of Z and v, the constants k_1 and k_2 have been calculated for the five energies of the four heaviest elements whose points are lying on a straight line. From these twenty values of k_1 and k_2 the mean values are found to be 1.20 and 1.35 with mean errors 10 % and 3 %, respectively. The value 1.35 is in agreement with an estimate of LINDHARD and SCHARFF[21], who find

$$L(x) = 1.36 \cdot x^{\frac{1}{2}} - 0.016 \cdot x^{\frac{3}{2}}$$

where $x = Z_2^{-1} \cdot v^2/v_0^2$.

As is well known, the factor Z_1^2 in the specific energy loss implies that the stopping of an α-particle is the same as that of a proton with an energy one quarter that of the α-particle. Therefore, the stopping power for protons at 1.5 MeV can be compared with earlier measurements for 6 MeV α-particles by MARSDEN and RICHARDSON, GEIGER, MANO and, ROSENBLUM. The values quoted by BETHE and LIVINGSTON[11] have been used. The values of the present investigation are, for all substances, 5—10 % lower than the average values of the four mentioned authors. It may be added that their results are obtained only relative to air, which demands an accurate knowledge of the stopping power of this substance. A general comparison of empirical ranges and specific energy loss is given in the paper of LINDHARD and SCHARFF[21], where it is found that the present results are not at variance with the recent accurate range curve in air by BETHE[22].

An extensive discussion of the fluctuations in the energy loss on a given path length ΔR has been given by BOHR[15]. It was shown by BOHR that for high velocities of the penetrating particle, where all electrons in the atoms contribute to the stopping, the average square of the fluctuations in energy loss is simply given by

$$\Omega^2 = 4\pi Z_1^2 \cdot e^4 \cdot Z_2 \cdot N \Delta R. \tag{5}$$

Since the energy loss can be written

$$\frac{\Delta E}{\Delta R} = \frac{4\pi Z_1^2 \cdot e^4 Z_2 \cdot N}{mv^2} L(x),$$

where $L(x)$ is the function shown in fig. 2, we may write, instead of (3),

$$u^2 = \frac{\Omega^2}{E \cdot \Delta E} = \frac{m}{M} \frac{2}{L(x)}, \tag{6}$$

where M is the proton mass.

For lower velocities of the particle, where only a part of the atomic electrons contribute to the energy loss, a reduction in straggling takes place. Here, we shall refer only to the calculations by LINDHARD and SCHARFF[21] who find:

$$u^2 = \frac{m}{M}. \tag{7}$$

The two formulas (6) and (7) should be joined smoothly.

In order to compare the measurements of the straggling with the theoretical estimate, the values of $u = \left(\frac{\Omega^2}{E \Delta E}\right)^{\frac{1}{2}}$ are plotted versus $x^{\frac{1}{2}} = Z_2^{-\frac{1}{2}} \cdot \frac{v}{v_0}$ in fig. 3. The points are found from the measured values of Ω, E and ΔE. The curves are drawn according to the value $\frac{m}{M} = 0.00055$ and to the semi-empirical value of $L(x) = 1.35 \cdot Z_2^{-\frac{1}{2}} \frac{v}{v_0}$ from equation (4).

In the case of mica, an effective atomic number has been found by interpolation between the values of the stopping powers of beryllium and aluminum. A value of $Z_2 = 10.5$ is used for mica.

From the spread of the points corresponding to the homogeneous mica foils the uncertainty in the values of Ω can be estimated to about $10\,^0/_0$. For the other substances the spread is a little greater. This may be due to small inhomogeneities in the foils

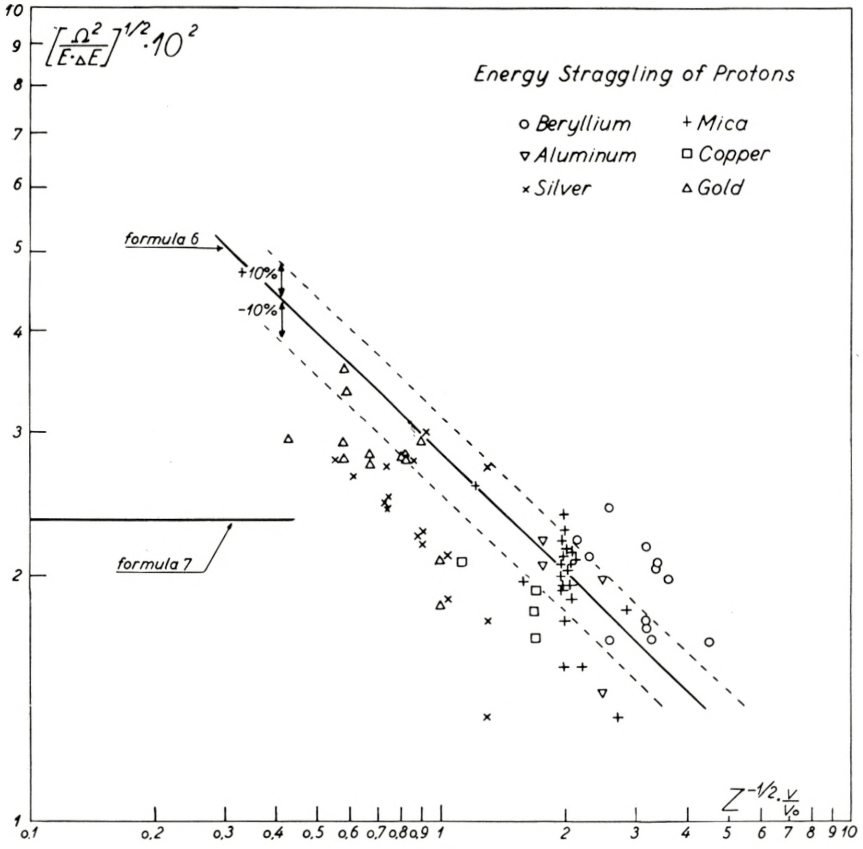

Fig. 3.

and sandwich targets. For the two heaviest substances, the values are about 1.3 times those corresponding to formula (7).

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After the completion of this paper, a communication has been received from Dr. S. A. Allison, Chicago, concerning some measurements of the stopping power, performed by Mr. David Kahn and to be published in the Physical Review. In case of the lightest and the heaviest elements, the agreement between this author's and our results is good; however, for medium elements, Kahn has found the stopping power to be higher than the present results. The largest deviations (20 per cent) are found for Copper, and they are much too large to be explained by impurities in the foils. The copper foils used in our investigation have been rolled or beaten, whereas Kahn's foils were prepared by evaporation. Since the method of foil preparation might be significant in explaining the difference in the results, a few remarks about the inhomogeneity of the foils may be useful.

When a commercial foil is inserted in the beam, contributions to the broadening of the resonance curve arise from 1) the width of the resonance curve without foil, 2) the straggling, and 3) the inhomogeneity of the foil. The width of the resonance curve without foil and that of the curve with foil can be found in the usual way. The contribution of the straggling can be estimated from the present measurements. Assuming the deviations to be added geometrically a standard deviation of the inhomogeneity of the foil can be found. In case of the 0,50 mg/cm^2 copper foil this quantity is 7 keV. The energy shift is found to 53 keV. The thickness is 0,50 $\pm$ 0,065 mg/cm^2, so that the inhomogeneity is 13 per cent. All the commercial foils showed an inhomogeneity of this magnitude.

A further communication has been received from Dr. J. N. Cooper, Ohio State University, whose results for copper foils, which have been prepared by an electrolytical process, lie between those of Kahn and those of the present author.

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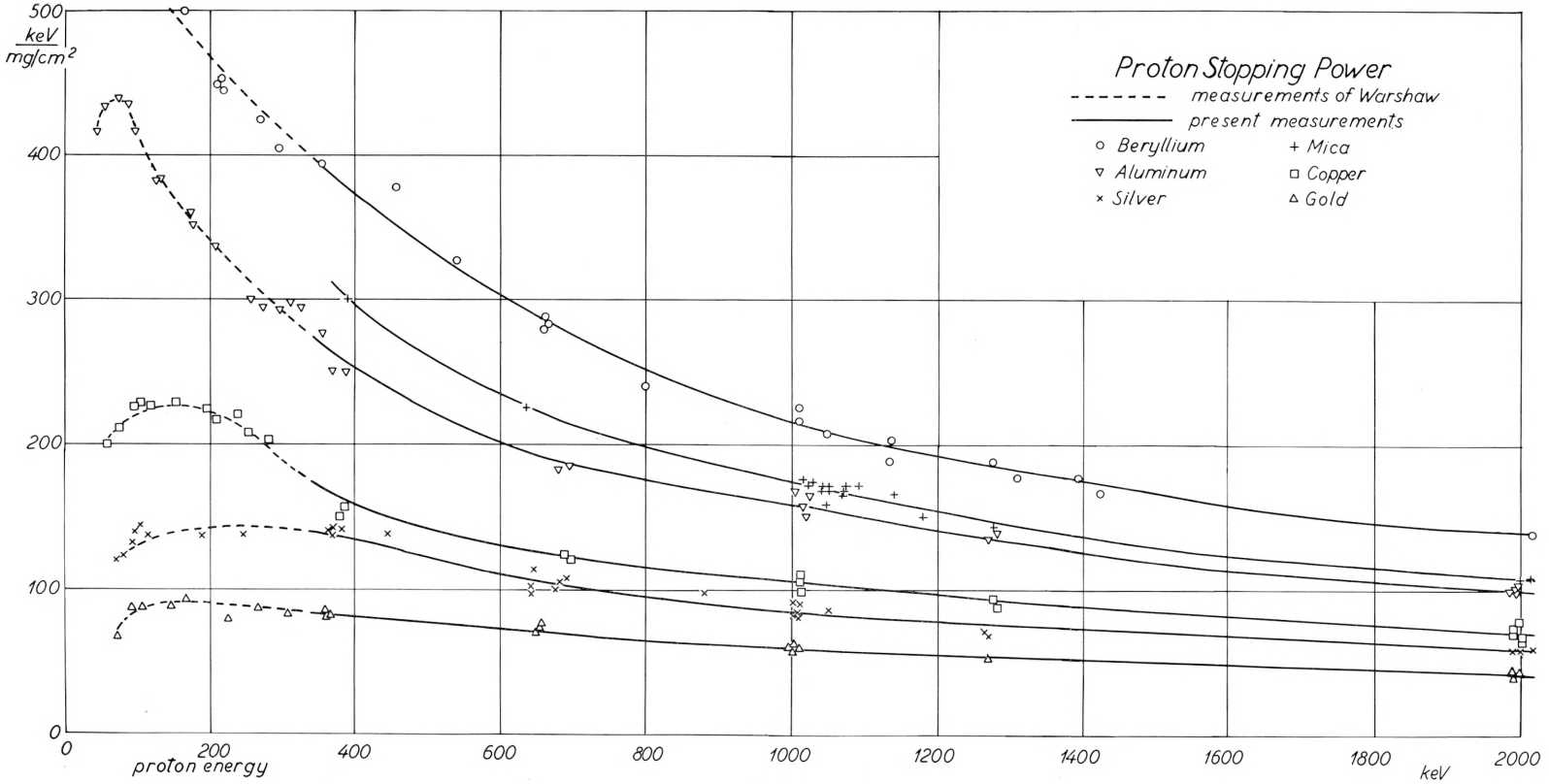
Proton Stopping Power
measurements of Warshaw
present measurements
Beryllium
Aluminum
Silver
Mica
Copper
Gold
500
keV
mg/cm²
400
300
200
100
0
0
200
400
600
800
1000
1200
1400
1600
1800
2000
proton energy
keV

Fig. 1.